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ΠGβ NORMAL SPACE IN INTUITIOITIC FUZZY TOPOLOGY

S. Jothimani[1], T. JenithaPremalatha[2]

[1]Department of Mathematics,
Government Arts and Science College, Coimbatore-641018, India
[2] Department of Mathematics, KPR Institute of Engineering and Technology,
Coimbatore -641407, India
e-mail: Jothimanis09@gmail.com,joel.jensi@gmail.com

Abstract. In this paper new notions of normality are introduced in intuitionistic fuzzy topological spaces using πgβ closed sets. In addition, it is established that this property is preserved by the continuous maps. The links between πgβ normality, almost πgβ normality and mildly πgβ normality are further investigated. In particular it is shown that that the addition of simple condition to the definition of β-normal space yields a property called πgβ normal space which is the weaker of β-normality.

Keywords: normality, fuzzy topological spaces, continuous maps.

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1. Introduction

Atanasov [4] generalized the idea of fuzzy sets and the concept of intuitionistic fuzzy sets was introduced .On the other hand Coker [6] introduced intuitionistic fuzzy topological spaces using intuitionistic fuzzy sets .In a topological space, several new classes of subsets can be obtained by repeatedly using interior and closure operators. Some of them are found to show remarkably interesting properties. A good number of research papers in recent years have been devoted to the study of these classes of sets and various notions related to them .Such class, the πgβ-closed sets, was introduced and studied in Intuitionistis fuzzy topological spaces in (Jenitha Premalatha and Jothimani , 2012). In this paper, we extend the notion of normality called πgβ -normality in Intuitionistic fuzzy topological spaces. The property of almost normality was introduced by the authors Singal and Arya [12] . The notion of mildly normal space was introduced by and Singal and Singal [13] independently. Mahmoud et al. [2] introduced the notion of β-normal spaces and obtained their characterizations and preservation theorems. The notion of quasi β-normal and mildly β–normal spaces were introduced by M. C. Sharma and Hamant Kumar [8].

The problem is that we will discuss here is what happens to these results when the normality is replaced with the πgβ normality in intuitionistic fuzzy topological spaces.

2. Preliminaries

Definition 2.1. [2] An intuitionistic fuzzy (IF) set A in X is an object having the form A = {<x, μ_A (x), ν_A(x)> / x∈ X} where μ_A(x) and ν_A(x) denote the degree of membership and non-membership respectively , and $0 \leq \mu_A (x) + \nu_A(x) \leq 1$.

Definition 2.2. [2] Let A and B be IFSs of the form A ={<x, μ_A (x), ν_A(x)> / x∈X} and B = { <x, μ_B (x), ν_B(x)> / x∈ X}. Then

i)A ⊆B if and only if $\mu_A (x) \leq \mu_B (x)$ and $\nu_A(x) \geq \nu_B(x)$ for all x ∈X

ii)A = B if and only if A ⊆ B and B ⊆ A

iii)A^c = {< x, ν_A(x), μ_A(x)>/ x ∈ X}

iv)A ∩ B = {<x, $\mu_A(x) \wedge \mu_B(x)$, $\nu_A(x) \vee \nu_B(x)$ > / x ∈ X}

v)A ∪ B = {<x, $\mu_A(x) \vee \mu_B(x)$, $\nu_A(x) \wedge \nu_B(x)$> / x ∈ X}.

Definition 2.3. [4] An intuitionistic fuzzy topology (IFT for short) on X is a family τ of IFSs in X satisfying the following axioms.

i)$0_\sim$, $1_\sim$ ∈τ

ii)$G_1 \cap G_2 \in \tau$ for any G_1, $G_2 \in \tau$

iii)$\cup G_i \in \tau$ for any family {G_i / i ∈ J} ⊆ τ.

Definition 2.4. [4] Let (X, τ) be an IFTS and A = <x, μ_A, ν_A> be an IFS in X. Then the intuitionistic fuzzy interior and intuitionistic fuzzy closure are defined by

IF-Int(A) = ∪{G / G is an IFOS in X and G ⊆ A}

IF-Cl(A) = ∩{K / K is an IFCS in X and A ⊆ K}

An IF subset A is said to be IF regular open [11] if A = IF-Int(IF-Cl(A)) The finite union of IF regular open sets is said to be IFπ-open [11]. The complement of a IFπ- open set is said to be IFπ- closed [11]. A is said to be IFβ-open [1] if A⊆IF-Cl(IF-Int(IF-Cl(A))). The family of all IFβ-open sets of X is denoted by IFβO(X). The complement of a IFβ–open set is said to be IFβ–closed [1]. The intersection of all IFβ–closed sets containing A is called IFβ–closure [2] of A, and is denoted by IFβCl(A). The IFβ-Interior [2] of A, denoted by IFβint(A), is defined as union of all IFβ-open sets contained in A. It is well known IFβ-Cl(A) = A∪IF-Int(IF-Cl(IF-Int(A))) and IF-β-Int(A)= A∩IF-Cl(IF-Int(IF-Cl(A))).

Definition 2.5. [13] An Intuitionistic fuzzy (IF) topological space (X,τ) is called mildly normal if for any two IF disjoint regularly closed subsets A and B of X, there exist two IF open disjoint subsets U and V of X such that A ⊆ U and B ⊆ V . i.e., any two IF disjoint regularly closed subsets are separated.

Definition 2.6. [12]An IF topological space (X,τ) is called almost normal if for any two disjoint IF closed subsets A and B of X, one of which is IF regularly closed, there exist two disjoint IF open subsets U and V of X such that A⊆U and B⊆V.

Definition 2.7. [16]An IF topological space (X,τ) is called quasi-normal if any two disjoint IFπ-closed subsets A and B of X there exist two IF-open disjoint subsets U and V of X such that A⊆U and B⊆V.

Definition 2.8. An IF space X is said to be IFβ –normal [2] if for every pair of disjoint IF-closed subsets A, B of X, there exist disjoint IFβ –open sets U, V of X such that A $\subseteq$U and B $\subseteq$ V.
Definition 2.9. An IF space X is said to be IFπβ –normal [8] if for every pair of disjoint IF-closed subsets A, B of X, one of which is IFπ–closed, there exist disjoint IFβ–open sets U, V of X such that A $\subseteq$U and B $\subseteq$V.
Definition 2.10. An IF subset A of a IF space X is said to be a IFβ–neighborhood [2] of x if there exists a IF β–open set U such that x $\in$ U $\subseteq$ A.
Definition 2.11. An IF function f : X $\to$Y is said to be
(a) IF-regular open [10] if f(U) is IF-regular open in Y for every open set U in X.
(b) IF π-continuous [7] if f^{-1}(F) is IFπ-closed in X for each IF closed set in Y.
(c) IF pre-β-closed [2] f(F) is IFβ-closed set in for every IF β-closed set Fin X.
(d) IFπgβ continuous [14] if f^{-1}(F) is IFπgβ closed in X for every IF closed set F in Y.
(e) IFπgβ -irresolute [14] if f^{-1}(F) is IFπgβ -closed in X for every IF -closed set F in Y.
(f) almost IFβ–irresolute [2] if for each x $\in$ X and IF β– neighborhood V of f(x) in Y, IFβ-Cl(f^{-1}(V)) is neighborhood of x in X.

3. πgβ Normal Spaces

In this section, we introduce the notion of IFπgβ -normal space and study some of its properties.
Definition 3.1. An IF space X is said to be IFπgβ -normal [15]) if for every pair of disjoint IFπgβ –closed subsets H and K of X, there exist disjoint IFβ–open sets U, V of X such that H $\subset$ U and K $\subset$ V.
Theorem3.1. For an IF topological space X, the following are equivalent:
(a) X is IFπgβ -normal.
(b) For every pair of disjoint IF πgβ -open subsets U and V of X whose union is X, there exist IF β-closed subsets G and H of X such that G$\subset$ U, H $\subset$V and G $\cup$H = X.
(c) For every IF πgβ -closed set A and every IF πgβ -open set B in X such that A $\subset$ B, there exists a IFβ-open subset V of X such that A $\subset$V $\subset$ IFβ-Cl(V) $\subset$ B.
(d) For every pair of disjoint IFπgβ -closed subsets A and B of X, there exists IF β-open subset V of X such that A $\subset$V and IF β-Cl(V) $\cap$B = 0~.
(e) For every pair of disjoint IFπgβ -closed subsets A and B of X, there exist IFβ-open subsets U and V of X such that A $\subset$U, B $\subset$V and IFβ-Cl(U)$\cap$IFβ-Cl(V) = 0~.
Proof. (a)$\Rightarrow$ (b) Let U and V be any IFπgβ –open subsets of a IFπgβ -normal space X such that U $\cup$ V = X. Then, X \ U and X \ V are disjoint IFπgβ –closed subsets of X. By IFπgβ –normality of X, there exist disjoint IFβ–open subsets U_1 and V_1 of X such that X \ U $\subset$ U_1 and X \ V $\subset$ V_1. Let G = X \ U_1 and H = X \ V_1. Then, G and H are IFβ–closed subsets in X such that G$\cup$H = X.

(b) ⇒ (c). Let A be a IF-πgβ –closed and B is IFπgβ- open subsets of X such that A ⊂B. Then, A∩(X\B) = 0~. Thus, X \ A and B are IFπgβ –open subsets of X such that (X\A)∪B=X. By the Part (b), there exist IF β–closed subsets G and H of X such that G ⊂ (X \ A), H ⊂ B and G ∪ H = X. Thus, we obtain that A ⊂ (X \ G) ⊂ H ⊂ B. Let V = X \ G. Then V is IFβ-open subset of X and IFβ-Cl(V)⊂H as H is IF β–closed set in X. Therefore, A ⊂ V ⊂IFβ-Cl(V) ⊂ B.

(c) ⇒ (d). Let A and B be disjoint IFπgβ -closed subset of X.

Then A⊂X \ B, where X \ B is IFπgβ -open. By the part (c), there exists a IFβ-open subset U of X such that A ⊂U⊂IFβ-C1(U)⊂ X \ B. Thus, IFβc1(U) ∩ B = 0~. (d) ⇒ (e). Let A and B be any disjoint IFπgβ -closed subset of X.

Then by the part (d), there exists a IF β-open set U containing A such that IFβcl(U) ∩B = 0~. Since IFβ-Cl(U) is IFπgβ - closed, then it is IF πgβ closed. Thus IFβ-Cl(U) and B are disjoint IFπgβ -closed subsets of X. Again by the part (d), there exists a IFβ-open set V in X such that B ⊂V and IFβcl(U) ∩IFβcl(V) =0~(e)⇒(a) Let A and B be any disjoint IFπgβ -closed subsets of X. Then by the part (e), there exist IFβ-open sets U and V such that A⊂U, B⊂V.and IFβ-Cl(U)∩IFβ-Cl(V) = 0~. Therefore, we obtain that U∩V= 0~.Hence X is IFπgβ -normal.

Lemma 3.1. (a) The image of IF β-open subset under an IF-open continuous function is IFβ-open.

(b)The inverse image of IF β-open subset under an open continuous function is IFβ-open subset.

Lemma 3.2.[15] The image of IF regular open subset under open and closed continuous function is IF regular open subset.

Lemma3.3. [15] The image of a IFβ-open subset under IF-open and IF-closed continuous function is IFβ-open.

Theorem3.2. If f : X →Y be an IF-open and IF-closed continuous bijection function and A be a IF πgβ -closed set in Y, then $f^{-1}(A)$ is IF πgβ-closed set in X.

Proof. Let A be an IFπgβ -closed subset of Y and U be any IF π-open subset of X such that $f^{-1}(A)$ ⊂U. Then by the Lemma 3.3, we have f (U) is IFπ–open subset of Y such that A ⊂f(U). Since A is IFπgβ -closed subset of Y and f(U) is IFπ–open set in Y. Thus IFβcl(A) ⊂U. By the Lemma 3.1 we obtain that $f^{-1}(A) \subset f^{-1}$(IFβ-cl(A)) ⊂U, where f^{-1}(IFβ-Cl(A)) is β- closed in X. This implies that IFβ-Cl($f^{-1}(A)$)⊂U. Therefore, $f^{-1}(A)$ is IFπgβ -closed set in X.

Theorem.3.3. If f: X →Y be an IF- open IF-closed bijective continuous function and X is IFπgβ -normal space. Then Y is also IFπgβ –normal.

Proof:.Let A and B be any disjoint IFπgβ - closed subsets of Y.Then by the Theorem3.2, $f^{-1}(A)$ and $f^{-1}(B)$ are disjoint of IFπgβ -closed subsets of X. By IFπgβ - normality of X, there exist IF β-open subsets U and V of X such that $f^{-1}(A) \subset U$, $f^{-1}(B) \subset V$ and U ∩V = 0~. By assumption, we have A ⊂f(U), B ⊂f (V) and f(U) ∩ f (V) = 0~. By the Lemma 3.1, f(U) and f(V) are disjoint IFβ-open subsets of Y such that A ⊂f(U) and B⊂ f(V). Hence, Y is IFπgβ –normal.

Definition 3.2. An IF function f :X→Y is called strongly IFπgβ- open if f (U)∈ IF-Πgβ- O(Y) for each U ∈IF-πgβ- O(X).

Definition 3.3. An IF function f : X → Y is called strongly IFπgβ- closed if f (U) ∈πgβ -C(Y) for each U ∈ πgβ- C(X).

Theorem 3.4. A IF function f : X → Y is strongly IFπgβ closed if and only if for each IF subset B in Y and for each IFπgβ-open set U in X containing f^{-1}(B), there exists a IFπgβ-open set V containing B such that f^{-1}(V) ⊂ U .

Proof. Suppose that f is strongly IFπgβ- closed. Let B be a IF subset of Y and U∈IFπgβ O(X) containing f^{-1}(B). Put V = Y \f (X\U), then V is a IFπgβ -open set of Y such that B ⊂ V and f^{-1}(V) ⊂ U .

Conversely, Let K be any IFπgβ -closed set of X . Then f^{-1}(Y \f (K)) ⊂ X \K and X \K ∈ πgβ- O(X). There exists a IF πgβ -open set V of Y such that Y \f (K) ⊂ V and f^{-1}(V) ⊂ X \K. Therefore, we have f (K) ⊃ Y \V and K ⊂ f^{-1}(Y \V). Hence, we obtain f (K) = Y \V and f (K) is IFπgβ -closed in Y. This shows that f is strongly IFπgβ -closed.

Theorem 3.5. If f : X → Y is a strongly IFπgβ –closed continuous function from a IFπgβ -normal space X onto an IF space Y , then Y is IFπgβ -normal.

Proof. Let S_1 and S_2 be disjoint IF-closed sets in Y . Then $f^{-1}(S_1)$ and $f^{-1}(S_2)$ are IF-closed sets. Since X is IFπgβ -normal, then there exist disjoint IFπgβ -open sets U and V such that $f^{-1}(S_1)$ ⊂ U and $f^{-1}(S_2)$ ⊂ V . By the previous theorem, there exist IFπgβ -open sets A and B such that S_1 ⊂ A, S_2 ⊂ B, f^{-1}(A) ⊂ U and f^{-1}(B) ⊂ V . Also, A and B are IF disjoint. Thus, Y is IF πgβ -normal.

Definition 3.4. A IF function f : X → Y is said to be almost IFπgβ- irresolute if for each IF point x in X and each IFπgβ neighborhood V of f (x), IFπgβ -Cl(f^{-1}(V)) is a IFπgβ neighborhood of x.

Lemma 3.4. Let f : X → Y be an IF function. Then f is almost IFπgβ irresolute if and only if f^{-1}(V) ⊂IF πgβ int(πgβ -Cl(f^{-1}(V))) for every V ∈ πgβ-O(Y).

Theorem 3.6. An IF function f : X → Y is almost IFπgβ irresolute if and only if f(IFπgβ -Cl(U)) ⊂IFπgβ Cl(f (U)) for every U ∈IF πgβ- O(X).

Proof. (⇒) : Let U ∈ IFπgβ- O(X). Suppose y ∈ IFπgβ-Cl(f (U)). Then there exists V ∈IFπgβ O(Y, y) such that V ∩ f (U) = 0~. Hence f^{-1}(V) ∩ U = 0~. Since U∈IFπgβ-O(X) we have IFπgβ -int(IFπgβ -Cl(f^{-1}(V))) ∩ IFπgβ -Cl(U) = 0~. Then by Lemma 3.4, f^{-1}(V) ∩ IFπgβ Cl(U) = 0~. and hence V ∩ f (πgβ -Cl(U)) = 0~. This implies that y∈IFπgβ Cl(U)).

Conversely, If V∈ IFπgβ O(Y), then M =X\IFπgβ-Cl(f^{-1}(V)) ∈F πgβ-O(X). By hypothesis, f(IFπgβ -Cl(M)) ⊂IFπgβ -Cl(f (M)) and hence X \ IFπgβ-int(IFπgβ -Cl(f^{-1}(V))) = IF πgβ-Cl(M)⊂IF f^{-1}(πgβ-IFCl(f (M))) ⊂ f^{-1}(IF πgβ-Cl(f (X \$f^{-1}$(V)))) ⊂ f^{-1}(IFπgβ -Cl(Y \V)) = f^{-1}(Y \V) =X \$f^{-1}$(V).

Therefore f^{-1}(V)⊂IFπgβ-int(IFπgβ -Cl(f^{-1}(V))).

By Lemma 3.4, f is almost IFπgβ -irresolute.

Theorem 3.7. If f : X → Y is a strongly IFπgβ -open continuous almost IFπgβ-irresolute function from a IFπgβ -normal space X onto a IF space Y , then Y is IFπgβ -normal.

Proof. Let A be an IF-closed subset of Y and B be an IF-open set containing A. Then by continuity of f, f^{-1}(A) is IF-closed and f^{-1}(B) is an IF open set of X such that

$f^{-1}(A) \subset f^{-1}(B)$. As X is IFπgβ -normal, there exists a IFπgβ open set U in X such that $f^{-1}(A) \subset U \subset$IF-πgβ-Cl(U) $\subset f^{-1}(B)$ by Theorem 3.1Then, f $(f^{-1}(A)) \subset$f (U)⊂f(IFπgβ -Cl(U)) ⊂ f $(f^{-1}(B))$. Since f is strongly IFπgβ- open almost IFπgβ -irresolute surjection, we obtain A ⊂ f (U) ⊂IF πgβ -Cl(f (U)) ⊂ B.
Then again by Theorem 1 the IF space Y is IFπgβ -normal.

4. Almost πgβ -normal spaces

Definition 4.1. An IF space X is said to be almost IFπgβ -normal if for each IF closed set A and each IF regular closed set B such that A∩B = 0~, there exist disjoint πgβ -open sets U and V such that A⊂U and B⊂V .
Theorem 4.1. For a IF space X the following statements are equivalent:
i)X is almost IFπgβ -normal,
ii) For every pair of sets U and V , one of which is IF-open and the other is IF regular open whose union is X , there exist IF πgβ -closed sets A and B such that A ⊂ U , B ⊂ V and A∪B = X,
iii)For every IF-closed set A and every IF-regular open set B containing A, there exists a IF- πgβ open set V such that A⊂V⊂ IFπgβ-Cl(V)⊂B.
Proof. (i) =⇒ (ii) : Let U be an IF open set and V be a IF regular open set in an almost IF πgβ -normal space X such that U ∪V = X . Then (X \U) is a IF-closed set and (X \V) is a IF regular closed set with (X \U) ∩(X \V) = 0~. By almost IFπgβ –normality of X, there exist disjoint IFπgβ open sets U_1 and V_1 such that X \ U ⊂ U_1 and X \ V ⊂ V_1. Let A = X \ U_1 and B = X \ V_1. Then A and B are IFπgβ closed sets such that A ⊂ U, B⊂V and A∪B=X .
(ii) ⇒ (iii) : Let A be a IF-closed set and B be a IF regular open set containing A. Then X \ A is IF open and B is IF regular open sets whose union is X . Then by (2), there exist IFπgβ closed sets M_1 and M_2 such that M_1 ⊂ X \ A and M_2 ⊂ B and M_1 ∪ M_2 = X . Then A ⊂ X \ M_1, X \ B ⊂ X \ M_2 and (X \ M_1) ∩ (X \ M_2) = 0~. Let U = X \ M_1 and V = X \ M_2.Then U and V are disjoint IFπgβ open sets such that A ⊂ U ⊂ X \ V ⊂ B. As X \ V is IF πgβ closed set, we have IFπgβ Cl(U)⊂X\V and A⊂U⊂IFπgβ -Cl(U)⊂B.
(iii) ⇒ (i):Let A_1 and A_2 be any two disjoint IF closed and IF regular closed sets, respectively Put D = X \ A_2, then A_2 ∩ D = 0~. A_1 ⊂ D where D is a IF regular open set. Then by (3), there exists a IFπgβ -open set U of X such thatA_1 ⊂ U ⊂IF πgβ-Cl(U) ⊂ D. It follows that A_2 ⊂ X \ IFπgβ -Cl(U) = V(say) , then V is IFπgβ -open and U ∩ V = 0~. Hence, A_1 and A_2 are separated by IFπgβ open sets U and V . Therefore X is almost IFπgβ -normal.
Definition:4.1. An IF function f: X → Y is called i) IF R-Map, [5] if $f^{-1}(V)$ is an IF regular open in X, for every IF regular open set V of Y.
ii) Completely Continuous [3]if $f^{-1}(V)$ is a IF regular open in X, for every IF open set V of Y.
Theorem 4.2. If f : X → Y is a IF continuous, strongly IFπgβ open , IFR-map and almost IFπgβ irresolute surjection from an almost IF πgβ -normal space X onto a IF space Y , then Y is almost IFπgβ -normal.

Proof. Similar to Theorem 3.7

Corollary 4.1: If f : X → Y is a completely IF continuous strongly IF πgβ open and almost IFπgβ irresolute surjection from an almost IF πgβ -normal space X onto a space Y , then Y is almost πgβ -normal.

5. Mildly IFπgβ -normal spaces

Definition 5.1. A IF space X is said to be mildly IFπgβ -normal if for every pair of disjoint IF regular closed sets A and B of X, there exist disjoint πgβ-open sets U and V such that A⊂U and B⊂ V .

Theorem 5.1. For a IF space X the following are equivalent:

(i) X is mildly IFπgβ -normal,

(ii)For every pair of IF regular open sets U and V whose union is X , there exist IFπgβ -closed sets G and H such that G⊂U , H⊂V and G∪H=X ,

(iii)For any IF regular closed set A and every IF regular open set B containing A, there exists a IF πgβ -open set U such that A⊂U⊂IF πgβ -Cl(U)⊂B,

(iv)For every pair of disjoint IF regular closed sets A and B, there exist IFπGβ - Open sets U and V such that A ⊂U , B⊂V and IFπgβ-Cl(U)∩IFπgβ -Cl(V)= 0~.

Proof. Similar to Theorem 3.1.

Theorem 5.2. If f : X → Y is a strongly IF πgβ -open IFR-map and almost IFπgβ irresolute function from a mildly IF πgβ -normal space X onto a IF space Y then Y is mildly IFπgβ -normal.

Proof. Let A be a IF regular closed set and B be a IF regular open set containing A. Then by IFR-map of f , $f^{-1}(A)$ is a IF regular closed set contained in the IF regular open set $f^{-1}(B)$. Since X is mildly IFπgβ normal, there exists a IFπgβ open set V such that $f^{-1}(A)$ ⊂V⊂IFπgβ -Cl(V)⊂$f^{-1}(B)$ by Theorem 8. As f is strongly IFπgβ open and an almost IFπgβ irresolute surjection, it implies, f (V)∈IFπgβ O(Y)and A ⊂f (V)⊂IF πgβ- Cl(f (V)) ⊂B. Hence Y is mildly IF πgβ -normal.

Theorem 5.3 If f : X → Y is IF R-map, strongly IFπgβ closed function from a mildly IFπgβ -normal space X onto a IF space Y , then Y is mildly IFπgβ -normal.

Proof. Similar to Theorem 3.5

Theorem 5.4. If f: X → Y is a continuous quasi IF- πgβ closed surjection and X is IF πgβ -normal, then Y is IF normal.

Proof. Let V_1 and V_2 be any disjoint IF closed sets of Y. Since f is IF continuous, $f^{-1}(V_1)$ and $f^{-1}(V_2)$ are disjoint IF closed sets of X . Since X is IFπgβ -normal, there exist disjoint U_1, U_2 ∈ πgβ O(X) such that $f^{-1}(V_i) \subset U_i$ for i = 1, 2. Put $W_i = Y - f(X - U_i)$, then V_i is IF open in Y , $V_i \subset W_i$ and $f^{-1}(W_i) \subset U_i$ for i = 1, 2. Since $U_1 \cap U_2$ = 0~, and f is IF surjective, we have $W_1 \cap W_2$ = 0~. This shows that Y is IF normal.

Theorem 5.5 Let f : X → Y be a closed πgβ -continuous injection. If Y IF πgβ - normal, then X is IFπgβ -normal.

Proof. Let N_1 and N_2 be disjoint IF closed sets of X. Since f is a IF Closed injection, f (N_1) and f (N_2) are disjoint IF closed sets of Y. By the IFπgβ-normality of Y , there exist disjoint V_1,V_2 ∈ IFβO(Y) such that f (N_i)⊂V_i for i = 1, 2. Since f is IFπgβ -continuous, $f^{-1}(V_1)$ and $f^{-1}(V_2)$ are disjoint IFπgβ -open sets of X and $N_i \subset f^{-1}(V_i)$ for i = 1, 2.

Now, put U_i = IF β-Int($f^{-1}(V_i)$) for i = 1, 2.Then, U_i ∈ IFπgβ O(X), $N_i \subset U_i$ and $U_1 \cap U_2$ = 0~. This shows that X is IFπgβ -normal

6. Preservation theorems

In this section we investigate preservation theorems concerning IFπgβ normal spaces in intuitionistic fuzzy topological spaces.

Theorem 6.1. If f : X → Y is an almost IFπgβ -closed injection and Y is mildly IFπgβ -normal respectively, then X is mildly IFπgβ normal.

Proof. Let A and B be any disjoint IF regular sets of X. Since f is an almost IF-πgβ -closed injection, f (A) and f (B) are disjoint IF regular πgβ-closed sets of Y . By the mild IFπgβ -normality of Y, there exist disjoint IF open sets U and V of Y such that (A)⊂U and f (B)⊂V. Now, put G = IF-Int (IF-Cl(U)) and H =IF- Int(IF-Cl(V)), then G and H are disjoint IF regular -open sets such that f (A) ⊂ G and (B) ⊂ H . Since f is almost IF πgβ -continuous, $f^{-1}(G)$ and $f^{-1}(H)$ are disjoint IFπgβ -open sets containing A and B, respectively. It follows from Theorem 5.1 that X is mildly IFπgβ -normal.

Lemma 6.1. A surjection f : X → Y is almost IF πgβ closed if and only if for each subset S of Y and each U ∈ IFπgβ -RO(X) containing $f^{-1}(S)$ there exists respectively an IFπgβ -open set V of Y such that S ⊂ V and $f^{-1}(V) \subset U$.

Proof. Necessity. Suppose that f is almost IFπgβ -closed.Let S be a subset of Y and let U∈IFπgβ-RO(X) contain $f^{-1}(S)$. Put V=Y\f (X\U), then V is a IFπgβ -open set of Y such that S⊂V and $f^{-1}(V) \subset U$

Sufficiency. Let F be any regular IFπgβ -closed set of X . Then f^{-1}(Y \f (F))⊂(X\ F) and (X \ F) ∈IF πgβ -RO(X). There exists a regular IF πgβ -open set V of Y such that (Y \ f (F))⊂V and $f^{-1}(V)$⊂(X\ F). Therefore, we have f (F)⊃Y \V and F⊂f^{-1} (Y \V). Hence f (F) = Y \ V, and f (F) is regular IFπgβ closed in Y. This shows that f is almost IF πgβ –regular -closed.

Theorem 6.2. If f: X → Y is a completely IFπgβ -continuous almost IFπgβ -closed surjection and X is mildly IFπgβ normal, then Y is IFπgβ -normal.

Proof. Let A and B be any disjoint IF closed sets of Y. Then $f^{-1}(A)$ and $f^{-1}(B)$ are disjoint IF πgβ closed sets of X . Since X is mildly IFπgβ -normal, there exist disjoint IF open sets U and V such that $f^{-1}(A) \subset U$ and $f^{-1}(B) \subset V$. Let G=IF-In (IF-Cl(U)) and H=IF-Int (IF-Cl(V)), then G and H are disjoint IF regular open sets such that $f^{-1}(A) \subset G$ and $f^{-1}(B) \subset H$. By Lemma 6.1, there exist IFπgβ -open sets K and L of Y such that A ⊂ K, B⊂ L, $f^{-1}(K) \subset G$ and $f^{-1}(L) \subset H$. Since G and H are disjoint, so are K and L,and L are IF πgβ -open, we obtain A ⊂IF-int(K), B ⊂IF- Int(L) and [IF-Int(K)∩IF-int(L)] = 0~. This shows that Y is IF πgβ -normal.

Corollary 6.1. If f: X → Y is a completely IFπgβ -continuous πgβ, closed surjection and X is mildly IFπgβ -normal, then Y is IF πgβ-normal.

Theorem 6.3: Let f : X → Y be an IFπgβ,IFR-map (almost πgβ -continuous) and almost IFπgβ -regular-closed surjection. If X is mildly IFπgβ -normal, then Y is mildly IFπgβ normal.

Proof. Let A and B be any disjoint regular IFπgβ -closed sets of Y. Then $f^{-1}(A)$ and $f^{-1}(B)$ are disjoint regular IFπgβ-closed or IF closed sets of X . Since X is respectively mildly IFπgβ normal there exist disjoint IF open sets U and V of X such that $f^{-1}(A) \subset U$ and $f^{-1}(B) \subset V$. Put G =IF-Int(IF-Cl(U)) and H = IF-Int (IF-Cl(V)), then G and H are disjoint IF regular open sets of X such that $f^{-1}(A) \subset G$ and $f^{-1}(B) \subset H$. By Theorem 6.2, there exist IF πgβ - open sets K and L of Y such that $A \subset K$, $B \subset L$, $f^{-1}(K) \subset G$ and $f^{-1}(L) \subset H$ .Since G and H are disjoint, so are K and L. It follows from Theorem 4.2 that Y is mildly IF πgβ normal.

Definition 6.1. A function f: $X \rightarrow Y$ is said to be IF π-irresolute [3] if $f^{-1}(F)$ is IFβ-closed in X for every IF β-closed set F in Y.

Theorem6.4. If f : $X \rightarrow Y$ is IF π-irresolute, IF pre β-closed and A is a IFπgβ -closed subset of X, then f(A) is IFπgβ -closed subset of Y.

Proof: Since f is IFπ-irresolute function, then $f^{-1}(U)$-IFπ-open in X. Since A is IFπgβ-closed set in X and $A \subset f^{-1}(U)$ then $IF\beta cl_X(A) \subset f^{-1}(U)$.This implies that $f(IF\beta cl_X(A)) \subset U$. Since f is IF pre β-closed and $IF\beta cl_X(A)$ is IFβ–closed set in X, then $f(IF\beta cl_X(A))$ is IFβ–closed inY. Thus, we have $IF\beta cl_Y(f(A)) \subset U$. Hence, f(A) is IFπgβ -closed subset of Y.

Corollary.6.2. If f: $X \rightarrow Y$ is IFπ-continuous, IF pre β-closed and A is a IFπgβ -closed subset of X, then f(A) is IFπgβ closed subset of Y.

Theorem 6.5. If f: $X \rightarrow Y$ is IFπ-irresolute, IF pre β-closed and IFβ-irresolute injection function from an IF space X to a IF πgβ –normal Y, then X is IFπgβ -normal.

Proof. Let A and B be any two disjoint IFπgβ closed subsets of X. By the Theorem 5.2 f(A) and f(B) are disjoint IFπgβ -closed subsets of Y. By IFπgβ -normality of Y, there exist disjoint IFβ- open subsets U and V of Y such that $f(A) \subset U$, $f(B) \subset V$ and $U \cap V = 0\sim$ Since f is IFβ-irresolute injection function, then $f^{-1}(U)$ and $f^{-1}(V)$ are disjoint IFβ-open sets in X such that $A \subset f^{-1}(U)$ and $B \subset f^{-1}(V)$. Hence X is IFπgβ -normal.

Corollary 6.3. If f : $X \rightarrow Y$ is IF π-continuous, IF pre β-closed and IFβ-irresolute injection function from a IF space X to a IF πgβ -normal Y, then X is IF πgβ -normal.

Lemma 6.1. If the IF bijection function f: $X \rightarrow Y$ is IF π–continuous and regular open, then f is πgβ -irresolute.

Theorem 6.6. If f : $X \rightarrow Y$ is πgβ –irresolute ,IF pre β-closed bijection function from a IFπgβ -normal space X to a IF space Y, then Y is IFπgβ -normal.

Proof. Let A and B be any two disjoint IFπgβ -closed subsets of Y. Since f is IFπgβ -irresolute, we have $f^{-1}(A)$ and $f^{-1}(B)$ are disjoint IF πgβ -closed subsets of X. By IFπgβ –normality of X, there exist disjoint IFβ-open sets U and V in X such that $f^{-1}(A) \subset U$, $f^{-1}(B) \subset V$ and $U \cap V = 0\sim$ Since f is IF pre β–open and IF bijection function, we have f(U) and f(V) are disjoint IFβ-open sets in Y such that $A \subset f(U)$, $B \subset f(V)$ and $f(U) \cap f(V)=0\sim$. Therefore, X is IFπgβ -normal.

Corollary 6.4. If f : $X \rightarrow Y$ is IFβ-continuous, IF regular open and IF pre β-open bijection function from a IFπgβ -normal space X to a IF space Y, then Y is IFπgβ –normal.

Theorem 6.7. If f: X → Y is a IF pre β–open, IF πgβ -irresolute and IF almost β– irresolute surjection function from a IF πgβ -normal space X onto a IF space Y, then Y is IFπgβ -normal.

Proof. Let A be a IF πgβ –closed subset of Y and B be a IFπgβ -open subset of Y such that A ⊂B. Since f is IFπgβ -irresolute, we obtain that $f^{-1}(A)$ is IFπgβ –closed in X and $f^{-1}(B)$ is IFπgβ -open in X such that $f^{-1}(A) \subset f$ -1(B). Since X is IFπgβ –normal, then by the Part (c) of the Theorem 3.1, there exists a IFβ– open set U of X such that $f^{-1}(A) \subset U \subset IF\beta cl_X(U) \subset f^{-1}(B)$. Then, $f(f^{-1}(A)) \subset f(U) \subset f(IF\beta cl_X(f(U)) \subset f(f^{-1}(B)$. Since f is IF pre β–open, IF almost β–irresolute surjection, we obtain that $A \subset f(U) \subset IF\beta cl_Y(f(U)) \subset B$ and f(U) is IFβ–open set in Y. Hence by the Theorem 3.6, we have Y is IFπgβ –normal.

Definition 6.2. An IF topological space (X, τ) is said to be Strongly IF πgβ -normal [9] if for each pair A, B⊆ X of disjoint IF πgβ -closed sets, there exist disjoint IFπgβ-open sets U and V of X such that A⊆ U and B⊆ V.

Theorem 6.8. For IF topological space (X, τ), the following are equivalent:

(i) (X,τ) is IFπgβ -normal.

(ii) For every IFπgβ -closed set A and every IFπgβ -open set U containing A, there is a IF β-clopen set V such that A⊆ V⊆ U.

Proof.

(i)⇒(ii). Let A be IFπgβ -closed and U be IFβ-open with A⊆ U. Now, we have A∩ (X \ U)= 0~ , hence there exist disjoint IFβ-open sets W_1 and W_2 such that A⊆ W_1 and X\U⊆ W_2. If V=IFβCl(W_1), then V is a IFβ-clopen set satisfying A⊆ V⊆ U.

(ii)⇒(i). Obvious.

Definition 6.3. A IF space (X, τ) is called weakly IF πgβ -normal if disjoint IFπgβ -closed set can be separated by disjoint closed sets.

Theorem 6.9. If f :(X, τ)→(Y, σ) is an IF injective ,IF- contra β-continuous always πgβ -closed function and (Y, σ) is weakly IF πgβ -normal, then (X,τ) is IF-πgβ -normal.

Proof. Suppose that A_1, A_2 ⊆ X are IFπgβ -closed and disjoint. Since f is always IFπgβ -closed and injective, f(A_1), f(A_2)⊆Y are IF πgβ -closed and disjoint. Since (Y, σ) is weakly IF πgβ -normal, f(A_1) and f(A_2) can be separated by disjoint IF closed sets B_1, B_2⊆ Y. More over as f is IF contra β-continuous, A_1 and A_2 can be separated by disjoint IF β-open sets $f^{-1}(B_1)$ and f^{-} (B_2). Thus (X,τ) is IFπgβ -normal.

7. Application of Normality

The Property of normality of a topological space is involved in a plenty of remarkable results. Especially, in proving Urysohn's Lemma and Tietze extension theorem. Most spaces encountered in mathematical analysis are normal Hausdorff spaces, or at least normal regular spaces. Another applications is a characterization of perfectly normal spaces, which leads to the characterization of hereditarily normal. To prove the given spaces are metrizable, it is enough to prove it as normal. An important example of a non-normal topology is given by the Zariski topology, which is used in algebraic geometry. The Sorgenfrey plane, is based on the phenomenon that the product of normal spaces is not necessarily normal. Normality in one of the separation axiom and it plays an important role in determining duals of spaces of continuous functions (in functional analysis). The property of normality is applied in the ideal theory of C*-algebras and related topics. Normal space is applied in summability theory and artificial neural networks.

8. Conclusion

The πgβ closed sets are used to introduce the concepts πgβ -normal space. Also, the characterization, the preservation & hereditary nature of πgβ -normal spaces have been framed and analyzed. In general, the entire content will be a successful tool for the researchers for finding the path to obtain the results in the context of normal spaces in bi topology and can be extended to Functional Analysis.

By the definitions stated above and in preliminaries. We have the following diagram:

normal → almost normal → mildly normal
↓ ↓ ↓
β-normal → almost β-normal → mildly β-normal
↓ ↓ ↓
πgβ-normal → almostπgβ- -normal → mildly πgβ—normal.

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